

Fast, Quality, Segmentation of Large Volumes - Isoperimetric Distance Trees

Siemens Corporate Research, Princeton USA
Department of Imaging and Visualization

Leo Grady
Leo.Grady@siemens.com



Motivation - Isoperimetric image segmentation on large volumes

Recently developed graph-based segmentation algorithm

Theory

Partition a weighted graph by minimizing the surface area to volume ratio

i.e., minimize the isoperimetric ratio: $h = \inf_{S} \frac{|\partial S|}{|S|}$

How to formalize?

Define a set of nodes (pixels) by an indicator vector:

$$x = \begin{cases} 1 & \text{if } v_i \in S, \\ 0 & \text{otherwise.} \end{cases}$$

How to define perimeter?

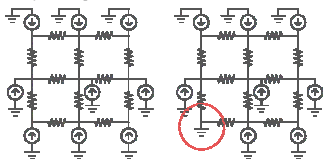
$$\partial S = \sum_{ij} w_{ij}(x_i - x_j)^2 = x^T L x$$

Volume more complicated - Depends on notion of inner product

$$|S| = x^T \mathbf{1}$$

$$|S| = x^T d$$

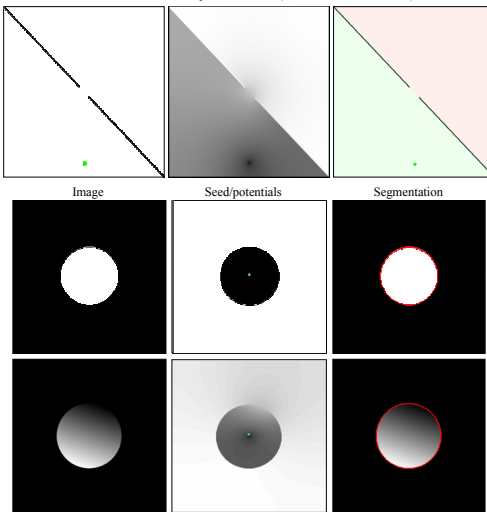
Relaxing binary (NP) problem, leads to an SPD linear system, which may be interpreted as a circuit simulation



Random walk interpretation also possible

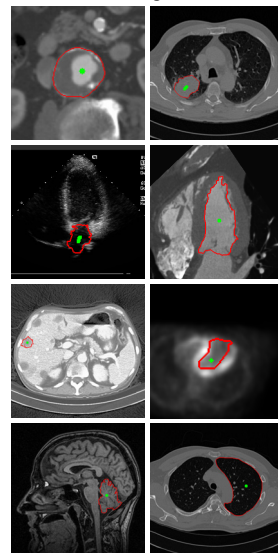
Properties

Weak boundary detection (4-connected lattice)



Also gives measure of segmentation quality and parallelizes easily

Examples



Problem: Want to run isoperimetric algorithm fast on large images/volumes

Solution: Use tree as underlying graph

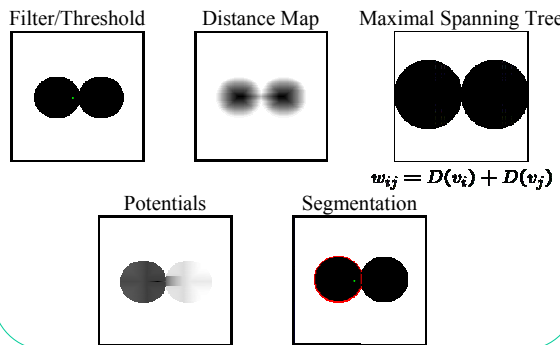
Effect on tree of Gaussian elimination

Original	1st elimination	2nd elimination	3rd elimination	Final elimination
$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	

Laplacian matrix Gaussian elimination

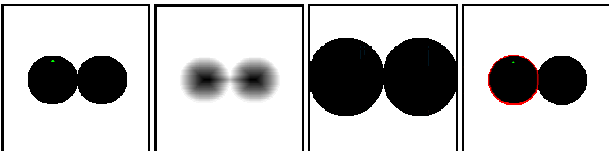
No fill! Isoperimetric algorithm runs in linear time!

How to define the tree?

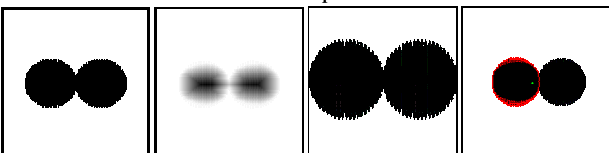


Compared to watersheds?

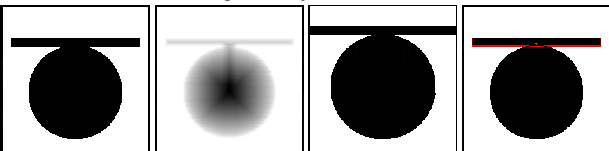
Watershed can handle basic case



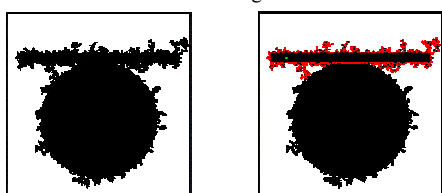
Noise - Multiple basins



Unable to separate objects in the same basin

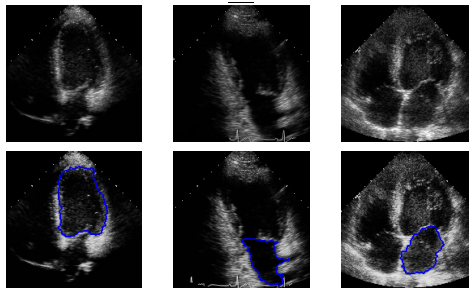


Noise and a single basin



Results

2D



3D

