Multi-Label Image Segmentation for Medical Applications Based on Graph-Theoretic Electrical Potentials

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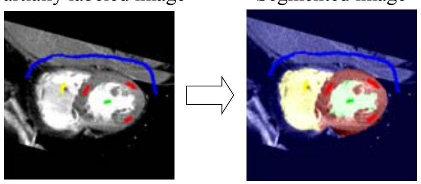
Problem

Partially labeled image | Segmented image

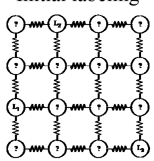


We want a general segmentation tool that is independent of modality, prior knowledge and segmentation task, and that allows easy (and fast) editing of the results.

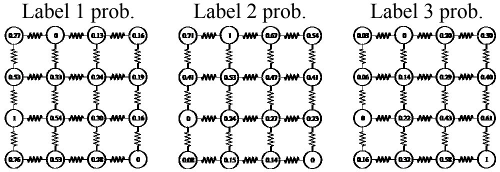
Computation

Situation exactly analogous to DC circuit steady-state

Initial labeling



Label 1 prob. | Label 2 prob. | Label 3 prob.



Labels – Unit voltage sources or grounds
Weights – Branch conductances
Probabilities – Steady-state potentials

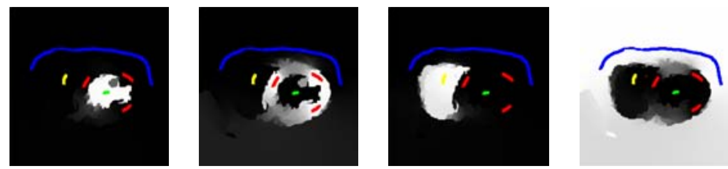
No simulation of random walks necessary!

Solve sparse system of SPD linear equations with a minor of the combinatorial Laplacian matrix

Main idea

What is the probability that a random walker starting at each pixel will first reach each of the different colors?

Green | Red | Yellow | Blue



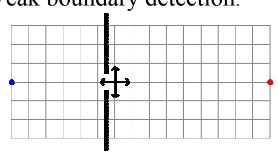
Probability = [1,0] mapped to [white, black]

Pixel is assigned to the color with the greatest probability

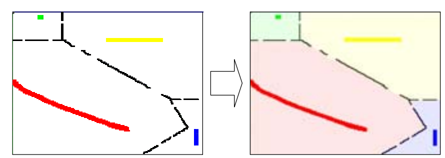
Random walker biased to avoid crossing intensity gradients

Why does this technique perform well?

Weak boundary detection:



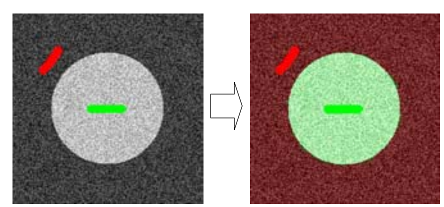
Random walker ¾ likely to stay on initial side of weak boundary



White image with incomplete black lines

Robust to noise:

As noise increases, expected value of the probabilities approach neutral (i.e., Voronoi-like) case



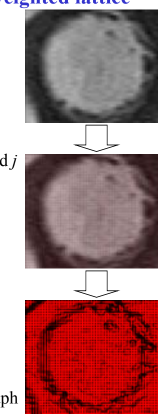
No texture or statistical techniques used

Connectivity:

Each segment is guaranteed to be connected to a seed point

Pixel values map to a weighted lattice

$$w_{ij} = e^{-\beta(I_i - I_j)^2}$$



w_{ij} weight between pixels i and j

I_i intensity at pixel i

β free parameter

β is the only free parameter in the entire algorithm.

Algorithm defined on arbitrary dimensional lattice or arbitrary graph

Examples

All results obtained with the same β!